LINEARLY DEP. & INDEP. SETS.

Let $V(F)$ be a V.S. over the field F & $S \subseteq V$.

Case 1: $S \to$ finite. Then S is said to be l.d./l.i according to the behavior of elts. of S (l.d./l.i)

Eg: $V(\mathbb{R}) = P_n(\mathbb{R})(\mathbb{R})$

$S_1 = \{1, x, x^2\} \to$ L.I.

$S_2 = \{1, x, 1+x\} \to$ L.D.

$\alpha_1 + \alpha_2 x + \alpha_3(1+x) = 0$

$\alpha_1 + \alpha_3 = 0 \Rightarrow \alpha_1 = -\alpha_3$

$\alpha_2 + \alpha_3 = 0 \Rightarrow \alpha_2 = -\alpha_3$

Case 2: S (INFINITE) is L.I. or L.D.

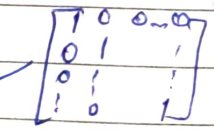
$\Rightarrow$ L.D. if $\exists$ atleast one finite subset of S which is L.D.

$\Rightarrow$ L.I. if every finite subset of S is L.I.

Eg: $V(\mathbb{R}) = \mathbb{R}^n(\mathbb{R}) = \{\langle a_n \rangle : a_n \in \mathbb{R}\}$

$S_1 = \{e_1, e_2, \dots\} \to$ L.I.

$S_2 = \{0, e_1, e_2 \dots\} \to$ L.D.



Quest: Let $V(F)$ be a V.S. $\{x_1, x_2\}$ is L.I. subset of V. Then W.O.T.F is/are true?

(A) $\{x_1+x_2, x_1-x_2\}$ is L.I. ✓

(B) $\{x_1+x_2, x_1-x_2\}$ is L.D. ✗

(C) $\{x_1, x_2, x_1+x_2\}$ is L.I. ✗

(D) $\{x_1, 2x_1\}$ is L.I. ✗

$\alpha(x_1+x_2) + \beta(x_1-x_2) = 0$

$\alpha + \beta = 0$

$\alpha - \beta = 0$

$\alpha x_1 + \beta x_2 + \gamma x_1 + \gamma x_2 = 0$

$\alpha + \gamma = 0$

$\beta + \gamma = 0$ $\Big\rangle$ $\alpha + \beta = 0$

$\alpha = -\beta$